EXERCISES

1 Find $f_1(x, y)$, $f_2(x, y)$, $f_{12}(x, y)$ if
(a) $f(x, y) = x^2 \log (x^2 + y^2)$
(b) $f(x, y) = x^y$.

2 With $f(x, y) = x^2y^3 - 2y$, find $f_1(x, y)$, $f_2(x, y)$, $f_2(2, 3)$, and $f_2(y, x)$.

3 Compute $\mathbf{D}f$ for each of the following functions at the given point:
(a) $f(x, y) = 3x^2y - xy^3 + 2$ at $(1, 2)$
(b) $f(u, v) = u \sin (uv)$ at $(\pi/4, 2)$
(c) $f(x, y, z) = x^2yz + 3xz^2$ at $(1, 2, -1)$.

4 (a) Let $f(x, y) = xy/(x^2 + y^2)$, with $f(0, 0) = 0$. Show that f_1 and f_2 exist everywhere, but that f is not of class C'.
(b) Does f have directional derivatives at the origin?
(c) Is f continuous at the origin?

5 Let a function f be defined in an open set D of the plane, and suppose that f_1 and f_2 are defined and bounded everywhere in D. Show that f is continuous in D.

6 Can you formulate and prove an analog for Rolle's theorem, for functions of two real variables?

7 Let f and g be of class C' in a compact set S, and let $f = g$ on bdy (S). Show that there must exist a point $p_0 \in S$ where $\mathbf{D}f(p_0) = \mathbf{D}g(p_0)$.

8 Find the derivative of $f(x, y, z) = xy^2 + yz$ at the point $(1, 1, 2)$ in the direction $(2/3, -1/3, 2/3)$.

9 Let $f(x, y) = xy$. Show that the direction of the gradient of f is always perpendicular to the level lines of f.

10 Show that each of the following obeys $\partial^2u/\partial x^2 + \partial^2u/\partial y^2 = 0$:
(a) $u = e^x \cos y$ (b) $u = \exp (x^2 - y^2) \sin (2xy)$

11 Let $f(x, y) = xy(x^2 - y^2)/(x^2 + y^2)$ with $f(0, 0) = 0$. Show that f is continuous everywhere, that f_1, f_2, f_{12}, and f_{21} exist everywhere, but $f_{12}(0, 0) \neq f_{21}(0, 0)$.

12 Find the directional derivative of $F(x, y, z) = xyz$ at $(1, 2, 3)$ in the direction from this point toward the point $(3, 1, 5)$.

13 If $F(x, y, z, w) = x^2y + xz - 2yw^2$, find the derivative of F at $(1, 1, -1, 1)$ in the direction $\beta = (4/7, -4/7, 1/7, -4/7)$.

14 Some economics students have been quoted as saying the following: $F(x_1, x_2, \ldots, x_n)$ is such that it does not change if you change only one variable, leaving the rest alone, but it does change if you make changes in two of them. What reaction would you give to such a statement?