χ_S is integrable over $S \subset \mathbb{R}^n$.
$\Longleftrightarrow$ Given $\varepsilon > 0$, there is a partition P over a rectangle R containing S such that upper sum minus lower sum $< \varepsilon$.

$$\Longleftrightarrow \sum_{R_{ij} \cap S \neq \emptyset} - \sum_{R_{ij} \subset S} < \varepsilon$$

$$\Longleftarrow \sum_{R_{ij} \cap \partial S \neq \emptyset} < \varepsilon$$

$\Longleftrightarrow$ Given $\varepsilon > 0$, S can be covered by finitely many boxes of total volume less than ε.
$\overset{\text{defn}}{\Longleftrightarrow}$ ∂S has zero content.
$\overset{\text{defn}}{\Longleftrightarrow}$ S is Jordan measurable.

The circled implication is your exercise to work out.

If S is Jordan measurable, we define the Jordan area as:

$$|S| = \text{area}(S) = \int_S \chi_S.$$