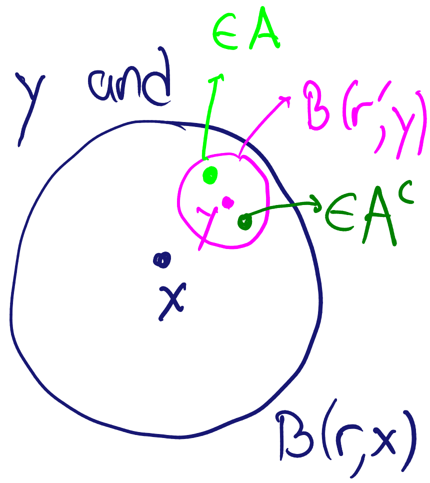
m231 - quiz #0

Let $A \subset \mathbb{R}^n$ be OPEN. Set $Y = \partial A$. Show $\partial Y = Y$

↳ I realized later that I forgot to write that on the board!

(1) $\partial Y \subseteq Y$: Let $x \in \partial Y$; in particular, $\forall\, r > 0$
$B(r,x) \cap \partial A \neq \emptyset$. Take such a y and any $B(r',y)$ in $B(r,x)$.
Since $y \in \partial A$, this small ball contains pts in A & in A^c.
Hence $x \in \partial A$ too.



(2) $Y \subseteq \partial Y$: Take any $x \in Y = \partial A$.
Any ball B centered at x contains a point $\alpha \in A$.
Since A is OPEN, $A \cap \partial A = \emptyset$. So $\alpha \notin \partial A$
Meanwhile B contains x and $x \in \partial A$.
Therefore $x \in \partial(\partial A)$.

REMARKS.

(1) $\partial\partial A \subseteq \partial A$: true regardless of openness of A.

(2) $\partial A \subseteq \partial\partial A$: not true in general. When A is open, (2) is true as I show above.

ex: $A = \mathbb{Q} \subset \mathbb{R}$: $\partial A = \mathbb{R} \not\supseteq \partial(\partial A) = \emptyset$

(3) A quick way to prove (1): Show $\partial A = \overline{A} \cap \overline{A^c}$.
If you show that, it follows that ∂A is closed because it's the intersection of two closed sets.
Since ∂A is closed, $\partial(\partial A) \subset \partial A$ (shown in class).