QUIZ #2

Let (X, d_X) & (Y, d_Y) be metric spaces with d_X being the discrete metric. When is a function $f: X \to Y$ continuous?

Answer: ALWAYS.

To show this you may employ any of the equivalent definitions for continuity.

either ① The ε-δ defn... To show continuity at $\alpha \in X$, given $\varepsilon > 0$, choose $\delta > 0$ such that $f(B_\delta(\alpha)) \subset B_\varepsilon(f(\alpha))$.
Simply choose any δ in $(0,1)$. In that case $B_\delta(\alpha)$ is the singleton $\{\alpha\}$ and $f(B_\delta(\alpha)) = f(\alpha) \in B_\varepsilon(f(\alpha))$.

OR ② Defn via sequences... For any $(x_j)_{j=1}^{\infty}$ in X with limit α, show $f(x_j) \xrightarrow[j\to\infty]{} f(\alpha)$.
Let (x_j) be such a sequence. Then (x_j) is ultimately the constant sequence (α) [We've proven this. Reprove it.]
Then $f(x_j) = f(\alpha) \xrightarrow[j\to\infty]{} f(\alpha)$.

OR ③ Every open in Y has open preimage in X...
Let $\underset{\neq \phi}{U \subset} Y$ be open. Then $f^{-1}(U)$ is open in X

lemma: (i) Any singleton is open in X. (Reprove!)
(ii) Any subset $A \subset X$ is open in X.

Pf. $A = \bigcup_{a \in A} \{a\}$. By (i), each $\{a\}$ is open.
Since union of open sets are open, A is open.