Spring 2024 Math 584 - Singularity Theory
Homework 1 - Manifolds
Due: 27/2/2024

GP: Guillemin & Pollack, Differential Topology
M1: Milnor, Topology from the Differentiable Viewpoint

1. Show that the two definitions for *a manifold in* $\mathbb{R}^n$ are equivalent.

   More precisely def 1 says that each point of the manifold X has a neighborhood in X that can be expressed as some level set of a function. def 2 is as in GP, page 3. Proving def 1$\Longrightarrow$def 2 is sort of straightforward. For def 2$\Rightarrow$def 1 you should solve essentially GP, p19, ex9.

2. Show that the orthogonal group $O(n)$ is a manifold in $\mathbb{R}^{n^2}$. (Do this at least for $n = 2$ and $n = 3$).

3. Show that the sphere in $\mathbb{R}^{n+1}$ with center 0 and radius K is diffeomorphic (as defined in GP, p3 or in M1, p1) to the unit sphere S^n.

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