Spring 2024 Math 584 - Singularity Theory
Homework 2 - Manifolds; Irreducible Varieties
Due: 7/3/2024

1. Let M be a manifold-with-boundary in $\mathbb{R}^k$ of dimension m. Show that
   (a) ∂M is an $(m-1)$-manifold.
   (b) $\partial(\partial M) = \emptyset$.

2. Let 0 be a regular value for $f : \mathbb{R}^n \to \mathbb{R}^k$ and $M = f^{-1}(0)$. Show that for $p \in M$, $T_pM = \ker Df(p)$.

3. (a) Prove REID, 3.5.(iv), concluding construction of Zariski topology.
   (b) Show that the following statements are equivalent for an algebraic V in $\mathbb{A}^n$: • V is irreducible; • Any two nonempty proper open subsets of V have nonempty intersection; • Any nonempty open subset of V is dense in V.

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