Spring 2024 Math 584 - Singularity Theory
Homework 7 - Puiseux-type numbers; Blow-up
Due: 22/5/2024

BK: Brieskorn-Knörrer, Plane Algebraic Curves
WALL: Wall, Singular Points of Plane Curves

1. Show that the three conventions for Puiseux numbers [(i) the Puiseux characteristics on WALL, p. 40; (ii) the characteristic Puiseux pairs on BK,p. 405-406; (iii) Zariski characteristic pairs on WALL, p. 63] are equivalent, i.e. they contain the same data.

2. If you blow-up an (oriented) algebraic surface S at a point, you obtain the connected sum of S and $\overline{\mathbb{C}P^2}$ (The latter is $\mathbb{C}P^2$ with reverse orientation). Moreover the connected sum is orientable. More generally, for any pair of oriented topological n-manifolds-with-boundary X and Y with $M = \partial X \cong \partial Y \neq \emptyset$, define the manifold-without-boundary

$$X \cup_f Y = \left(X \coprod Y \right) / x \sim f(x)$$

where $\coprod$ denotes the disjoint union, f is an orientation reversing homeomorphism from ∂X to ∂Y, and the RHS has the quotient topology. Show that $X \cup_f Y$ orientable, it has a natural orientation induced from X and Y. How do you use the fact that f is orientation reversing?

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