Important theorems to be proven in math338 – Complex Analysis:

CB Theorem 21: If $f = u + iv$ is differentiable at z_0 then partials of u and v exist at z_0 and they satisfy the CR equations.

Cauchy-Goursat: If f is analytic over Ω then its integral is zero along any closed contour in Ω.

Cauchy Integral Theorem: For f analytic over Ω, C any closed contour in Ω and z_0 inside C, the integral of $f/(z - z_0)$ along C equals $2\pi i f(z_0)$.

CB Theorem 52.1: If a function is analytic at a point, then its derivatives of all orders are analytic there too.

Liouville's Theorem: A bounded entire function is constant.

BN Theorem 6.11: If f is entire and if $f(z) \to \infty$ as $z \to \infty$, then f is a polynomial.

Fundamental Theorem of Algebra: Every non-constant polynomial with complex coefficients has a zero in $\mathbb{C}$.

Gauss-Lucas Theorem: The zeroes of the derivative of any polynomial lie within the convex hull of the zeroes of the polynomial.

Uniqueness Theorem: Suppose f is analytic over Ω and that $f(z_n) = 0$ where $\{z_n\}$ is a sequence of distinct points and $z_n \to z_0 \in \Omega$. Then $f \equiv 0$ in Ω.

Maximum-Modulus Theorem: A non-constant analytic function in a region does not have any interior maximum points.

Open Mapping Theorem: The image of an open set under a nonconstant analytic mapping is an open set.

Morera's Theorem: Let f be a continuous function on a region Ω. If its integral is 0 along the boundary curve of any rectangle in Ω then f is analytic over Ω.

Cauchy Residue Theorem: Let C be a positively oriented simple closed contour. If a function f is analytic inside and on C except for a finite number of singular points $z_k (k = 1, 2, \ldots, n)$ inside C, then

$$\int_C f(z)dz = 2\pi i \sum_{k=1}^{n} (\text{residue of } f \text{ at } z_k).$$